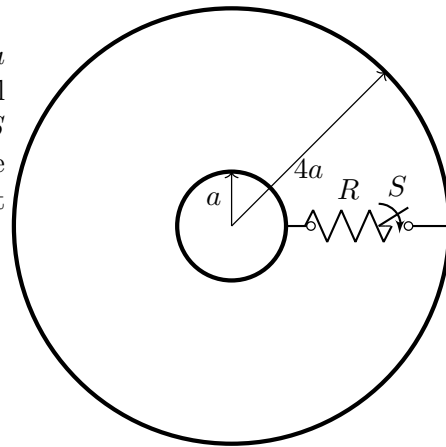


1. Two concentric conducting spherical shells of radius a and of radius $4a$ form a spherical capacitor. Initially the net charge on the inner shell is Q while the outer shell is uncharged. At time $t = 0$ the switch S connects the inner shell to the outer shell over a resistor of resistance R . Assume that the resistance is small enough so that it does not change the capacitance of the system.



- [5 Pts.] (a) What is the final net charge on the outer shell?
- [10 Pts.] (b) What is the capacitance of the system?
- [10 Pts.] (c) At what time would the charges on two shells be equal?

a) All charge flows outside $\rightarrow Q$

b) Gauss' law $|\mathbf{E}_{in}| = \frac{Q_{in}}{4\pi\epsilon_0 r^2} \Rightarrow$

$$\begin{aligned}\Delta V &= \frac{Q_{in}}{4\pi\epsilon_0 r^2} \left(\frac{1}{a} - \frac{1}{4a} \right) \\ &= \frac{3}{16\pi\epsilon_0} \frac{Q_{in}}{a},\end{aligned}$$

So,

$$C = \frac{Q}{\Delta V} = \frac{Q_{in}}{\frac{3d}{16\pi\epsilon_0} \frac{Q_{in}}{a}} = \frac{16\pi}{3} \epsilon_0 a$$

c)

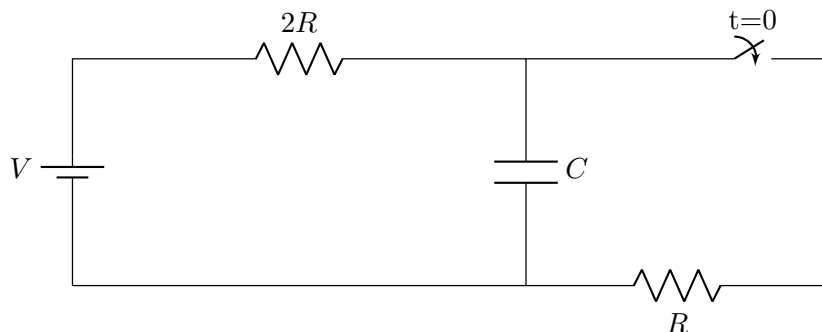
$$\begin{aligned}I &= \frac{\Delta V}{R} = \frac{3}{16\pi\epsilon_0 R a} Q_{in} = \frac{1}{RC} Q_{in} \\ \frac{dQ_{in}}{dt} &= -I = -\frac{1}{RC} Q_{in} \Rightarrow Q_{in} = Q e^{-t/RC}\end{aligned}$$

We want

$$\begin{aligned}Q(t) &= \frac{Q}{2} = Q e^{-t_e/RC} \\ RC \ln 2 &= t_e \\ t_e &= \frac{16\pi\epsilon_0}{3} R a \ln 2\end{aligned}$$

2. An ideal voltage source V is connected to a capacitor of capacitance C over a resistor with resistance $2R$. Another resistor of resistance R is placed parallel to the capacitor, but separated from the circuit by a switch which has been open for a long time. At time $t = 0$ the switch is closed.

- [5 Pts.] (a) What is the charge on the capacitor just before the switch is closed?
- [5 Pts.] (b) What is the charge on the capacitor a long (infinite!) time after the switch is closed?
- [10 Pts.] (c) Find the charge on the capacitor as a function of time $Q(t)$ for all $t > 0$.
- [5 Pts.] (d) What is the time constant of the circuit with the switch closed.

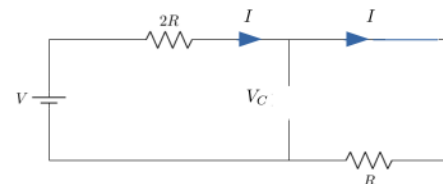


a)

$$V_C = V \Rightarrow Q(t = 0^-) = VC$$

b)

$$V_C = \frac{V}{3} \Rightarrow Q(t \rightarrow \infty) = \frac{VC}{3}$$



c)

$$i_1 = \frac{Q(t)}{RC}, \quad i_2 = \frac{\frac{Q(t)}{C} - V}{2R}$$

$$\frac{dQ}{dt} = -i_1 - i_2 = -\frac{Q(t)}{RC} - \frac{Q(t)}{2RC} + \frac{V}{2R}$$

shift the variable $q = Q - VC/3 \Rightarrow \frac{dQ}{dt} = \frac{dq}{dt}$

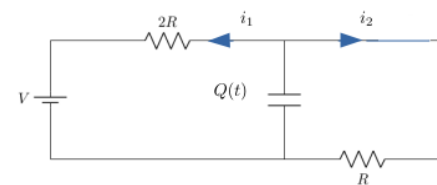
$$\frac{dq}{dt} = -\frac{3}{2RC}q \Rightarrow q(t) = q(0)e^{-\frac{3t}{2RC}}, \text{ and } q(0) = \frac{2VC}{3}$$

So,

$$Q(t) = \frac{2}{3}VCe^{-\frac{t}{\tau}} + \frac{VC}{3}$$

d) Time constant

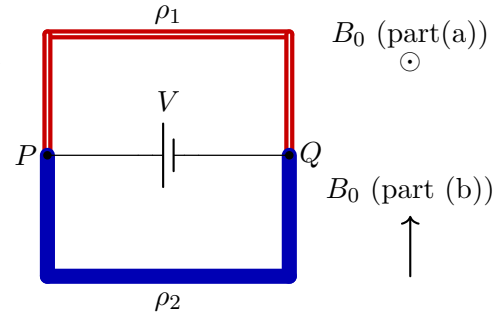
$$\tau = \frac{2RC}{3}$$



3. A conducting square loop of side L is made up of two wires. The upper half is made of a material with resistivity ρ_1 , while the lower half's material has resistivity ρ_2 . Both wires have cross sectional area A . The connecting points of the wires, P and Q , which are at the middle of opposing sides, are connected to a DC voltage source V with wires of negligible resistance as shown in the figure. You do not need to consider forces on these wires.

[12 Pts.] (a) What is the net force (direction and magnitude) on the loop if there is a magnetic field B_0 coming out of the page?

[13 Pts.] (b) What is the magnitude of the torque with respect to the axis passing through P and Q if there is a magnetic field B_0 in the plane of the loop directed perpendicular to that axis?



a)

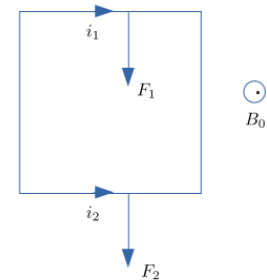
$$R_1 = 2L \frac{\rho_1}{A} \Rightarrow i_1 = \frac{V}{R_1} = \frac{VA}{2L\rho_1}$$

$$R_2 = 2L \frac{\rho_2}{A} \Rightarrow i_2 = \frac{V}{R_2} = \frac{VA}{2L\rho_2}$$

$$|\vec{F}_1| = B_0 L i_1 = \frac{B_0 V A}{2\rho_1} \quad \text{downward}$$

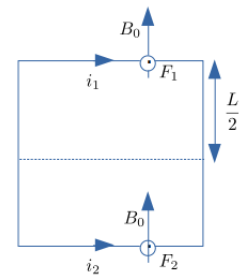
$$|\vec{F}_2| = B_0 L i_2 = \frac{B_0 V A}{2\rho_2} \quad \text{downward} \Rightarrow |\vec{F}_T| = \frac{B_0 V A}{2} \frac{\rho_1 + \rho_2}{\rho_1 \rho_2} \quad \text{downward}$$

Side forces are canceling each other



b) Both $\vec{F}_1$ and $\vec{F}_2$ are $\odot$, $\vec{\tau} = \vec{r} \times \vec{F}$
They are at opposite sides of the axis

$$|\vec{\tau}| = \frac{L}{4} B_0 V A \left| \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \right| = \frac{B_0 V A L}{4\rho_1 \rho_2} |\rho_2 - \rho_1|$$



4. A particle with charge q and mass m is moving in a region of space where there is a uniform magnetic field given by $\mathbf{B} = B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$. At time $t = 0$ the velocity of the particle is $\mathbf{v}_0 = v_0 \hat{\mathbf{i}}$.

[8 Pts.]

(a) Find the force on the particle at $t = 0$.

[5 Pts.]

(b) What is the period of the motion?

[12 Pts.]

(c) Find the components of the velocity vector of the particle for $t > 0$.

a)

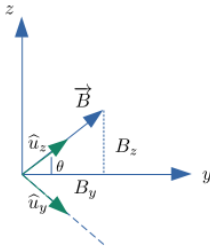
$$\vec{F} = q\vec{v} \times \vec{B} = qv_0[(\hat{i} \times \hat{j})B_y + (\hat{i} \times \hat{k})B_z] = -qv_0B_z\hat{j} + qv_0B_y\hat{k}$$

b) $|\vec{B}| = \sqrt{B_y^2 + B_z^2} = B_0$, then $|\vec{F}| = qvB_0$

$$|\vec{F}| = m|\vec{a}| = m\frac{v^2}{R} = qvB_0 \Rightarrow v = \frac{qB_0}{m}R$$

$$\text{period } T = \frac{2\pi R}{v} = \frac{2\pi m}{q\sqrt{B_y^2 + B_z^2}}$$

c) Define $\cos \theta = \frac{B_y}{B_0}$ and $\sin \theta = \frac{B_z}{B_0}$ and rotate the coordinates



$$\hat{u}_z = \cos \theta \hat{j} + \sin \theta \hat{k}$$

$$\hat{u}_y = \sin \theta \hat{j} - \cos \theta \hat{k} \Rightarrow \vec{B} = B_0 \hat{u}_z$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$

$$\frac{d\vec{v}}{dt} = \frac{qB_0}{m}(\vec{v} \times \hat{u}_z)$$

$$\text{Then, } \frac{dv_x}{dt} = \frac{qB_0}{m}v'_y \quad \frac{dv'_y}{dt} = -\frac{qB_0}{m}v_x$$

$$\text{where } \hat{v} = v_x \hat{i} + v'_y \hat{u}_y + v'_z \hat{u}_z$$

Then

$$\hat{v} = v_0 \cos \left(\frac{qB_0}{m}t \right) \hat{i} - v_0 \sin \left(\frac{qB_0}{m}t \right) \sin \theta \hat{j} + v_0 \sin \left(\frac{qB_0}{m}t \right) \cos \theta \hat{k}$$